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Computational algorithm PAOR for time-fractional diffusion equations

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Abstract. We deal with the application of an unconditionally implicit finite difference approximation equation of the one-dimensional linear time fractional diffusion equations (TFDE's) via the Caputo's time fractional derivative. Based on this implicit approximation equation, the corresponding linear system can be generated in which its coefficient matrix is large scale and sparse. To speed up the convergence rate in solving the linear system iteratively, we construct the corresponding preconditioned linear system. Then we formulate and implement the Preconditioned AOR (PAOR) iterative method for solving the generated linear system. One example of the problem is presented to illustrate the effectiveness of PAOR method. The numerical results of this study show that the proposed iterative method is superior to PSOR and PGS, GS iterative method.

1. Introduction

Based on previous studies in [1,2,3,4] many successful mathematical models, which are based on fractional partial derivative equations, have been developed. Following to that, there are several methods used to solve these models. For example, researchers have proposed finite difference methods such as explicit and implicit [5,6,7]. Also it is pointed out that the explicit methods are conditionally stable. Therefore, we discretize the time-fractional diffusion equation (TFDE's) via the implicit finite difference discretization scheme and Caputo's fractional partial derivative of order α in order to derive a Caputo's implicit finite difference approximation equation. This approximation equation leads a tridiagonal linear system. Due to the properties of the coefficient matrix of the linear system which is sparse and large scale, iterative methods are the alternative option for efficient solutions. As far as iterative methods are concerned, it can be observed that many researchers such as Young [8], Hackbusch [9] and Saad [10] have proposed and discussed several families of iterative methods. Among the existing iterative methods, the preconditioned iterative methods Hoang-hao [11], Gunawardena [12] have been widely accepted to be one of the efficient methods for solving linear systems.

Because of the advantages of these iterative methods, the aim of this paper is to construct and investigate the effectiveness of the Preconditioned AOR (PAOR) iterative method for solving time fractional diffusion equations (TFDE's) based on the Caputo's implicit finite difference approximation equation. To investigate the effectiveness of the PAOR method, we also implement the PSOR and PGS,GS iterative methods being used a control method.

To show the effectiveness of PAOR method, let time fractional diffusion equations (TFDe's) be defined as

$$\frac{\partial^{\alpha} U(x,t)}{\partial^{\alpha}} = a(x)\frac{\partial^{2} U(x,t)}{\partial x^{2}} + b(x)\frac{\partial U(x,t)}{\partial x} + c(x)U(x,t) \quad (1)$$

where *a(x), b(x)* and *c(x)* are known functions or constants, whereas α is a parameter which refers to the fractional order of time derivative.

2. Preliminaries

In this section, the space-fractional diffusion equation (1) is solved. In order to find solution in Eq.(1), let us define $h = \frac{\ell}{m+1}$, where, $m=n+1$ is positive even integer. By implementing definition (2) we obtain

To constructing the Caputo's implicit finite difference approximation equation of Eq.(1), the following are some basic definitions for fractional derivative theory which are used in this paper.

Definition 1.[8] The Riemann-Liouville fractional integral operator, J^{α} of order-α is defined as

$$J^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha} f(t) dt \;, \; \alpha > 0 \; x > 0 \tag{2}$$

Definition 2.[8] The Caputo's fractional partial derivative operator, D^{α} of order -α is defined as

$$D^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x \frac{f^{(m)}(t)}{(x-t)^{\alpha-m+1}} dt \;, \; \alpha > 0 \tag{3}$$

with $m - 1 < \alpha \leq m$, $m \in N$, $x > 0$

To obtain the numerical solution of Eq.(1) with Dirichlet boundary conditions, firstly we derive an implicit finite difference approximation equation based on the Caputo's derivative definition and the non-local fractional derivative operator. This implicit approximation equation can be categoried as unconditionally stable scheme. A discretize approximation to the time fractional derivative in Eq. (1) by using Caputo's fractional partial derivative of order α, is defined as[10,11]

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{1}{\Gamma(n-1)} \int_0^{\infty} \frac{\partial u(x-s)}{\partial t} (t-s)^{-\alpha} ds, \quad t > 0, \quad 0 < \alpha < 1 \tag{4}$$

3. Approximation For Fractional Diffusion Equation

In this paper, FSAOR, HSAOR and QSAOR iterative methods will be applied to solve linear system generated from the discretization of the problem in Eq.(1) as shown in Eq.(10). To derive the formulation of both proposed methods, let the coefficient matrix A in Eq.(10) be expressed as

From Eq. (4), the formulation of Caputo's fractional partial derivative of the first order approximation method is given as

$$D_t^{\alpha} U_{i,n} \cong \sigma_{\alpha,k} \sum_{j=1}^{n} \omega_j^{(\alpha)} \left(U_{i,n-j+1} - U_{i,n-j} \right) \tag{5}$$

and we have the following expressions

$$\sigma_{\alpha,k} = \frac{1}{\Gamma(1-\alpha)(1-\alpha)k^{\alpha}} \quad \text{and} \quad \omega_j^{(\alpha)} = j^{1-\alpha} - (j-1)^{1-\alpha}.$$

Before discretizing Eq.(1), let the solution domain of the problem be partitioned uniformly. To do this, we consider some positive integers *m* and *n* in which the grid sizes in space and time directions for the finite difference algorithm are defined as $h = \Delta x = \frac{\gamma - 0}{m}$ and $k = \Delta t = \frac{T}{n}$ respectively. Based on these grid sizes, we construct the uniformly grid network of the solution domain where the grid points in the space interval $[0, \gamma]$ are indicated as the numbers $x_i = ih$, $i = 0,1,2,\ldots, m$ and the grid points in the time interval $[0, T]$ are labeled $t_j = jk$, $j = 0,1,2,\ldots, n$. Then the values of the function $U(x,t)$ at the grid points are denoted as $U_{i,j} = U(x_i, t_j)$.

By using Eq. (5) and the implicit finite difference discretization scheme, the Caputo's implicit finite difference approximation equation of Problem (1) to the grid point centered at $(x_i, t_j) = (ih, nk)$ is given as

$$\sigma_{\alpha,k} \sum_{j=1}^{n} \omega_j^{(\alpha)} \left(U_{i,n-j+1} - U_{i,n-j} \right) = a_i \frac{1}{h^2} \left(U_{i-1,n} - 2U_{i,n} + U_{i+1,n} \right) + b_i \frac{1}{2h} \left(U_{i+1,n} - U_{i-1,n} \right) + c_i U_{i,n}, \quad (6)$$

for $i=1,2...,m-1$.

Based on Eq. (6), this approximation equation is known as the fully implicit finite difference approximation equation which is consistent first order accuracy in time and second order in space. Basically, the approximation equation (6) can be rewritten based on the specified time level. For instance, we have for $n \geq 2$:

$$\sigma_{\alpha,k} \sum_{j=1}^{n} \omega_j^{(\alpha)} \left(U_{i,n-j+1} - U_{i,n-j} \right) = \left(\frac{a_i}{h^2} - \frac{b_i}{2h} \right) U_{i-1,n} + \left(c_i - \frac{2a_i}{h^2} \right) U_{i,n} + \left(\frac{a_i}{h^2} + \frac{b_i}{2h} \right) U_{i+1,n}, \quad (7a)$$

$$\therefore \sigma_{\alpha,k} \sum_{j=1}^{n} \omega_j^{(\alpha)} \left(U_{i,n-j+1} - U_{i,n-j} \right) = p_i U_{i-1,n} + q_i U_{i,n} + r_i U_{i+1,n},$$

where

$$p_i = \frac{a_i}{h^2} - \frac{b_i}{2h}, \quad q_i = c_i - \frac{2a_i}{h^2}, \quad r_i = \frac{a_i}{h^2} + \frac{b_i}{2h}.$$

Also, we get for $n = 1$,

$$-p_i U_{i-1,1} + q_i^* U_{i,1} - r_i U_{i+1,1} = f_{i,1}, \quad i = 1,2,...,m-1 \quad (7b)$$

where

$$\omega_j^{(\alpha)} = 1, \; q_i^* = \sigma_{\alpha,k} - q_i, \; f_{i,1} = \sigma_{\alpha,k} U_{i,1}.$$

From, Eq. (7b), it can be seen that the tridiagonal linear system can be constructed in matrix form as

$$A \underset{\sim}{U} = \underset{\sim}{f} \quad (8)$$

where

$$A = \begin{bmatrix} q_1^* & -r_1 & & & & \\ -p_2 & q_2^* & -r_2 & & & \\ & -p_3 & q_3^* & -r_3 & & \\ & & \ddots & \ddots & & \\ & & & -p_{m-2} & q_{m-2}^* & -r_{m-2} \\ & & & & -p_{m-1} & q_{m-1}^* \end{bmatrix}_{(m-1)\times(m-1)}$$

$$\underset{\sim}{U} = \begin{bmatrix} U_{11} & U_{21} & U_{31} & \cdots & U_{m-2,1} & U_{m-1,1} \end{bmatrix}^T$$

$$\underset{\sim}{f} = \begin{bmatrix} U_{11} + p_1 U_{01} & U_{21} & U_{31} & \cdots & U_{m-2,1} & U_{m-1,1} + p_{m-1} U_{m,1} \end{bmatrix}^T.$$

4. Preconditioned Aor Iterative Method

In relation to the tridiagonal linear system in Eq. (8), it is clear that the characteristics of its coefficient matrix are large scale and sparse. As mentioned in Section 1, many researchers have discussed various iterative methods such as Young [8], Hackbusch [9], Saad [10], Sunarto *et.al* [13]. To obtain numerical solutions of the tridiagonal linear system (8), we consider the Preconditioned AOR (PAOR) iterative method [11, 14], which is the most known and widely using for solving any linear systems.

Before applying the PAOR iterative method, we need to transform the original linear system (8) into the preconditioned linear system

$$A^* \underset{\sim}{x} = \underset{\sim}{f}^* \quad (9)$$

where,

$$A^* = PAP^T,$$

$$\underset{\sim}{f}^* = P \underset{\sim}{f}, \quad \underset{\sim}{U} = P^T \underset{\sim}{x}.$$

Actually, the matrix P is called a preconditioned matrix and defined as [12]
$P = I + S$ where

$$S = \begin{bmatrix} 0 & -r_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r_3 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & -r_{m-1} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{(m-1)x(m-1)}$$

and the matrix I is an identical matrix. To formulate PAOR method, let the coefficient matrix A^* in (8) be expressed as summation of the three matrices

$$A^* = D - L - V \tag{10}$$

where D, L and V are diagonal, lower triangular and upper triangular matrices respectively. By using Eq. (9) and (10) , the formulation of PAOR iterative method can be defined generally as [11,14]

$$\underset{\sim}{x}^{(k+1)} = (D - \omega L)^{-1}[\beta V + (\beta - \omega)D + (1-\beta)D]\underset{\sim}{x}^{(k)} + \beta(D - \omega L)^{-1} f \tag{11}$$

where $\underset{\sim}{x}^{(k+1)}$ represents an unknown vector at $(k+1)^{th}$ iteration. The implementation of the PAOR iterative method can be described in Algorithm 1.

Algorithm 1: PAOR method

i. Initialize $\tilde{U} \leftarrow 0$ and $\varepsilon \leftarrow 10^{-10}$.

ii. For $j = 1,2,\ldots,n$ Implement

For $i = 1,2,\ldots,m-1$ calculate

$$\underset{\sim}{x}^{(k+1)} = (D - \omega L)^{-1}[\beta V + (\beta - \omega)D + (1-\beta)D]\underset{\sim}{x}^{(k)} + \beta(D - \omega L)^{-1} f$$

$$\underset{\sim}{U}^{(k+1)} = P^T \underset{\sim}{x}^{(k+1)}$$

Convergence test. If the convergence criterion i.e $\left\| \underset{\sim}{U}^{(k+1)} - \underset{\sim}{U}^{(k)} \right\| \le \varepsilon = 10^{-10}$ is satisfied, go to Step (iii). Otherwise go back to Step (a).

Iii Display approximate solutions.

5. Numerical Experiment

With approximation Eq.(7), we consider one example of the time fractional diffusion equation to test the effectiveness of the Gauss-Seidel (GS), Preconditioned Gauss-Seidel (PGS), Preconditioned SOR (PSOR) and PAOR iterative methods. In order to compare the effectiveness of these two proposed iterative methods, three criteria have been considered such as number of iterations, execution time (in seconds) and maximum absolute error at three different values of $\alpha = 0.25$, $\alpha = 0.50$ and $\alpha = 0.75$. For implementation of both iterative schemes, the convergence test considered the tolerance error, which is fixed as $\varepsilon = 10^{-10}$.

Let us consider the time fractional initial boundary value problem be given as [15]

$$\frac{\partial^\alpha U(x,t)}{\partial t^\alpha} = \frac{\partial^2 U(x,t)}{\partial x^2}, \quad 0 < \alpha \le 1, 0 \le x \le \gamma, \quad t > 0 \tag{12}$$

where the boundary conditions are stated in fractional terms

$$U(0,t) = \frac{2kt^\alpha}{\Gamma(\alpha+1)}, \; U(\ell,t) = \ell^2 + \frac{2kt^\alpha}{\Gamma(\alpha+1)}, \tag{13}$$

and the initial condition

$$U(x,0) = x^2 \tag{14}$$

All results of numerical experiments for Problem (12), obtained from implementation of GS, PGS, PSOR and PAOR iterative methods are recorded in Table 1 at different values of mesh sizes, m = 128, 256, 512, 1024, and 2048.

6. Conclusions

For the numerical solution of the time fractional diffusion problems, the paper presents the derivation of the Caputo's implicit finite difference approximation equations in which this approximation equation leads a linear system. From observation of all experimental results by imposing the PGS,PSOR and PAOR iterative methods, it is obvious at $\alpha = 0.25$ that number of iterations have declined approximately by 64.87-99.79% corresponds to the PAOR iterative method compared with the GS method. Again in terms of execution time, implementations of PAOR method are much faster about 4.95-98.97% than the PSOR and PGS method. It means that the PAOR method requires the least amount for number of iterations and computational time at $\alpha = 0.25$ as compared with PSOR and PGS iterative methods. Based on the accuracy of both iterative methods, it can be concluded that their numerical solutions are in good agreement.

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Table 1. Comparison of number iterations (K), the execution time (seconds) and maximum errors for the iterative methods using example at $\alpha = 0.25, 0.50, 0.75$

		α = 0.25			α = 0.50			α = 0.75		
M	**Method**	**K**	**Time**	**Max Error**	**K**	**Time**	**Max Error**	**K**	**Time**	**Max Error**
128	**GS**	21017	37.73	9.9e-5	13601	5.92	9.8e-5	6695	2.94	1.3e-4
	PGS	7292	35.86	9.9e-5	4715	2.23	9.8e-5	2319	1.93	1.3e-4
	PSOR	281	2.24	9.9e-5	229	1.95	9.8e-5	164	1.63	1.3e-4
	PAOR	280	1.12	9.9e-5	225	1.50	9.8e-5	160	1.59	1.3e-4
256	**GS**	77231	343.63	1.0e-4	50095	42.17	9.9e-5	24732	20.70	1.3e-4
	PGS	26884	261.56	9.9e-5	17417	16.68	9.8e-5	8585	12.37	1.3e-4
	PSOR	1428	16.90	9.9e-5	1171	12.61	9.8e-5	814	8.90	1.3e-4
	PAOR	1100	12.44	9.9e-5	950	10.75	9.8e-5	713	8.13	1.3e-4
512	**GS**	281598	2747.34	1.2e-4	183181	339.85	1.0e-4	90783	166.75	1.3e-4
	PGS	98422	1916.28	1.0e-4	63298	123.01	9.9e-5	31619	62.78	1.3e-4
	PSOR	5524	113.86	9.9e-5	4520	91.37	9.8e-5	11695	61.98	1.3e-4
	PAOR	4397	92.58	9.9e-5	3754	78.34	9.8e-5	2780	59.09	1.3e-4
1024	**GS**	1017140	68285.36	1.0e-4	663971	2454.53	1.0e-5	330622	1209.39	1.4e-4
	PGS	357258	14064.44	1.4e-4	232784	1007.47	1.0e-5	115617	820.93	1.3e-4
	PSOR	20574	817.59	9.9e-5	16842	662.23	9.8e-5	11695	456.23	1.3e-4
	PAOR	16487	699.81	9.9e-5	14058	607.00	9.8e-5	10394	429.58	1.3e-4
2048	**GS**	3631638	58914.30	1.3e-4	2380946	17795.25	1.3e-4	1192528	8794.26	1.7e-4
	PGS	121156	4104.17	1.3e-4	19153.0	3239.84	13e-5	112899	1305.5	1.3e-4
	PSOR	75580	3043.59	1.3e-4	61941	2894.7	9.9e-5	43070	337.1	1.3e-4
	PAOR	56289	3002.21	1.3e-4	46535	2870.12	9.9e-5	33819	305.2	1.3e-4

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