

[Fractal Fract] Manuscript ID: fractalfract-1290842 - Submission Received

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 23 Juni 2021 20.00

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 Kepada: Andang Sunarto <andang99@gmail.com>

 Cc: Praveen Agarwal <goyal.praveen2011@gmail.com>, Jumat Sulaiman <jumat@ums.edu.my>, Jackel Vui Lung

 Chew <jackelchew93@ums.edu.my>, Shaher Momani <S.Momani@ju.edu.jo>

Dear Dr. Sunarto,

Thank you very much for uploading the following manuscript to the MDPI submission system. One of our editors will be in touch with you soon.

Journal name: Fractal and Fractional Manuscript ID: fractalfract-1290842 Type of manuscript: Article Title: Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation Authors: Andang Sunarto *, Praveen Agarwal, Jumat Sulaiman, Jackel Vui Lung Chew *, Shaher Momani Received: 23 June 2021 E-mails: andang99@gmail.com, goyal.praveen2011@gmail.com, jumat@ums.edu.my, jackelchew93@ums.edu.my, S.Momani@ju.edu.jo

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If you have any questions, please do not hesitate to contact the Fractal Fract editorial office at fractalfract@mdpi.com

Kind regards,

Fractal Fract Editorial Office St. Alban-Anlage 66, 4052 Basel, Switzerland E-Mail: fractalfract@mdpi.com Tel. +41 61 683 77 34 Fax: +41 61 302 89 18

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[Fractal Fract] Manuscript ID: fractalfract-1290842 - Assistant Editor Assigned

Eric Yan <eric.yan@mdpi.com>24 Juni 2021 09.47Balas Ke: eric.yan@mdpi.comKepada: Andang Sunarto <andang99@gmail.com>Cc: Eric Yan <eric.yan@mdpi.com>, Praveen Agarwal <goyal.praveen2011@gmail.com>, Jumat Sulaiman<jumat@ums.edu.my>, Jackel Vui Lung Chew <jackelchew93@ums.edu.my>, Shaher Momani<S.Momani@ju.edu.jo>, Fractal Fract Editorial Office <fractalfract@mdpi.com>

Dear Dr. Sunarto,

Your manuscript has been assigned to Eric Yan for further processing who will act as a point of contact for any questions related to your paper.

Journal: Fractal and Fractional Manuscript ID: fractalfract-1290842 Title: Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation Authors: Andang Sunarto *, Praveen Agarwal, Jumat Sulaiman, Jackel Vui Lung Chew *, Shaher Momani

Received: 23 June 2021 E-mails: andang99@gmail.com, goyal.praveen2011@gmail.com, jumat@ums.edu.my jackelchew93@ums.edu.my, S.Momani@ju.edu.jo

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Best regards, Mr. Eric Yan Assistant Editor E-Mail: eric.yan@mdpi.com

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[Fractal Fract] Manuscript ID: fractalfract-1290842 - Major Revisions

3 pesan

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19 Juli 2021 10.49

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Thank you again for your manuscript submission:

Manuscript ID: fractalfract-1290842 Type of manuscript: Article Title: Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation Authors: Andang Sunarto *, Praveen Agarwal, Jumat Sulaiman, Jackel Vui Lung Chew *, Shaher Momani Received: 23 June 2021 E-mails: andang99@gmail.com, goyal.praveen2011@gmail.com, jumat@ums.edu.my, jackelchew93@ums.edu.my, S.Momani@ju.edu.jo

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Please use the version of your manuscript found at the above link for your revisions.

(I) Any revisions to the manuscript should be marked up using the "Track Changes" function if you are using MS Word/LaTeX, such that any changes can be easily viewed by the editors and reviewers.

(II) Please provide a cover letter to explain, point by point, the details of the revisions to the manuscript and your responses to the referees' comments.

(III) If you found it impossible to address certain comments in the review reports, please include an explanation in your rebuttal.

(IV) The revised version will be sent to the editors and reviewers.

If one of the referees has suggested that your manuscript should undergo extensive English revisions, please address this issue during revision. We propose that you use one of the editing services listed at https://www.mdpi.com/authors/english or have your manuscript checked by a native English-speaking colleague.

Do not hesitate to contact us if you have any questions regarding the revision of your manuscript. We look forward to hearing from you soon.

Kind regards,

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Mr. Eric Yan Assistant Editor E-Mail: eric.yan@mdpi.com

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Fractal Fract Editorial Office St. Alban-Anlage 66, 4052 Basel, Switzerland E-Mail: fractalfract@mdpi.com http://www.mdpi.com/journal/fractalfract

Dear Editor,

With this cover letter, we submit the revised manuscript (**fractalfract-1290842**) entitled "Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation" for publication in Fractal and Fractional.

We want to thank reviewers for the careful and constructive reviews. Based on the comments from the reviewers, we have made changes to the manuscript, which are detailed below.

Reply to the comments by Reviewer 1

1. What about the stability analysis of the new numerical approximation?

Reply: Thank you for the comment. In this paper, Theorem 1 is given to state the unconditional stability of the new numerical approximation call the quarter-sweep implicit finite difference approximation via Caputo's time-fractional operator. The presentation of the proof has been revised to make the stability analysis clearer to the readers. Please see Proof of Theorem 1 in Section 4.

2. Authors have considered two different linear fractional diffusion problems to test the efficiency and accuracy of the numerical method. But it is expected to solve numerically a nonlinear diffusion type problem of fractional order?

Reply: Thank you for the comment. Solving a nonlinear fractional diffusion equation using the quarter-sweep finite difference scheme with Caputo's fractional operator is our current ongoing research. The numerical experiment with a nonlinear fractional diffusion needs a thorough investigation, and hence the publication will take some time.

3. Authors should correct the grammatical and typo errors placed in the paper.

Reply: Thank you for the comment. The grammatical and typo errors in this paper has been identified as much as possible and fixed accordingly.

4. Authors should improve the introduction by including the recent development within the frame of the numerical scheme in the Caputo fractional order sense and their illustrative applications with the help of recently published papers.

Reply: Thank you for the comment. The introduction part of the paper has been revised accordingly. Please see Para. 1 in Section 1.

5. They should extend the concluding remarks section so that it provides and covers all the finding of the paper and future direction.

Reply: Thank you for the comment. The concluding remark section has been revised accordingly. Please see Section 8.

6. What makes the proposed numerical scheme suitable for this unique task? What new development to the proposed method have the authors added (compared to the existing approaches)? These points should be clarified.

Reply: Thank you for the comment. The answers to the above questions can be referred to in Para. 2 in Section 1.

7. *Have you employed any assumptions in your models? Please explain briefly.*

Reply: Thank you for the comment. This paper assumes the solutions exist in the restricted rectangular finite-difference solution domain. The solutions are verified after the numerical solutions converged at a desired tolerance errors in the iteration cycle. Besides that, the examples of the time-fractional diffusion equation are selected due to the existence of exact solutions.

8. More physical interpretations should be given. Please provide corresponding explanations of the figures in terms of their physical meanings and the pointing out the novelty of the paper. How do figures support your scheme?

Reply: Thank you for the comment. This paper investigates the efficiency of a quarter-sweep scheme via Caputo's time-fractional approximation, which is evaluated based on the number of iterations and execution time at several different mesh sizes. The figures illustrate the improvement in efficiency by the proposed numerical method against the existing works. The discussion section has been expanded to explain the interpretation made based on the collected results. Please see Para. 3 and 4 in Section 7.

9. In figures, the differences (advantages or disadvantages of the methods) should be clarified.

Reply: Thank you for the comment. Please see Para. 3 and 4 in Section 7.

Reply to the comments by Reviewer 2

- 1. It is not clear to me how equation (19) is obtained. Give detailed explanations for that.
- 2. *Give detailed explanations how equation (23) is obtained.*
- *3. Stability result should be established for the proposed method defined by equation (15).*

Reply: Thank you for these comments. The stability analysis section has been revised accordingly. Please see Section 4.

4. The numerical order of convergence of proposed method is not provided in the paper. It is suggested to include it in the paper.

Reply: Thank you for the comment. The convergence analysis section has been added. Please see Section 5.

5. Authors solved very simple numerical examples (special case of equation (3)) using the proposed method to show the applicability of present method. It is suggested to solve more challenging diffusion problems in order to demonstrate the wider applicability of the method.

Reply: Thank you for the comment. The numerical solution of a more challenging diffusion problem will be investigated further and will be published nearly in the future.

6. Literature review on recently developed computational techniques for solving timefractional diffusion equation is very poor.

Reply: Thank you for the comment. The literature review in the introduction part of the paper has been revised accordingly. Please see Para. 1 and 2 in Section 1.

7. Please check the manuscript carefully for typos and grammar errors.

Reply: Thank you for the comment. The grammatical errors and typos have been fixed accordingly.

We appreciate the comments from the reviewers. Thank you for reviewing our manuscript.

Sincerely, Andang Sunarto



[Fractal Fract] Manuscript ID: fractalfract-1290842 - Revised Version Received 2 pesan

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28 Juli 2021 09.08

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Manuscript ID: fractalfract-1290842 Type of manuscript: Article Title: Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation Authors: Andang Sunarto *, Praveen Agarwal, Jumat Sulaiman, Jackel Vui Lung Chew *, Shaher Momani Received: 23 June 2021 E-mails: andang99@gmail.com, goyal.praveen2011@gmail.com, jumat@ums.edu.my, jackelchew93@ums.edu.my, S.Momani@ju.edu.jo

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Kind regards,

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[Fractal Fract] Manuscript ID: fractalfract-1290842 - Minor Revisions

2 pesan

Fractal Fract Editorial Office <fractalfract@mdpi.com> Balas Ke: eric.yan@mdpi.com 2 Agustus 2021 10.44

Kepada: Andang Sunarto <andang99@gmail.com>

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Dear Dr. Sunarto,

Thank you again for your manuscript submission:

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 (IV) Please provide a short cover letter detailing your changes for the editors' and referees' approval.

If one of the referees has suggested that your manuscript should undergo extensive English revisions, please address this issue during revision. We propose that you use one of the editing services listed at https://www.mdpi.com/authors/english or have your manuscript checked by a native English-speaking colleague.

Please do not hesitate to contact us if you have any questions regarding the revision of your manuscript or if you need more time. We look forward to hearing from you soon.

Kind regards, Mr. Eric Yan Assistant Editor E-Mail: eric.yan@mdpi.com

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Andang Sunarto <andang99@gmail.com> Kepada: eric.yan@mdpi.com

Greeting Prof Eric Thank you for your information

Andang Sunarto Indonesia [Kutipan teks disembunyikan] 2 Agustus 2021 10.49

Dear Reviewer 1,

We want to thank you for the careful and constructive reviews on our manuscript (**fractalfract-1290842**) entitled ''Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation'' for publication in Fractal and Fractional.

Based on your comments, we have made further changes to the manuscript, which are detailed below.

Reply to the comments by Reviewer 1

1. Authors should improve the introduction by including the recent development within the frame of the numerical scheme in the Caputo fractional order sense and their illustrative applications with the help of recently published papers.

Reply: Thank you for the comment. The introduction part of the manuscript has been revised accordingly by including several recent papers related to the frame of the numerical scheme in Caputo sense with their applications. Please see Page 2, Line 48-77.

2. They should extend the concluding remarks section so that it provides and covers all the finding of the paper and future direction.

Reply: Thank you for the comment. The conclusion part of the manuscript has been extended by highlighting the findings of the paper and future direction. Please see Page 13, Line 397 to Page 14, Line 419.

3. What makes the proposed numerical scheme suitable for this unique task? What new development to the proposed method have the authors added (compared to the existing approaches)? These points should be clarified.

Reply: Thank you for the comment. The above questions have been answered accordingly. Please see Page 2, Line 83-95.

Dear Reviewer 2,

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- *3. Stability result should be established for the proposed method defined by equation (15).*

Reply: Thank you for these comments. The stability analysis section has been revised accordingly. Please see Section 4.

4. The numerical order of convergence of proposed method is not provided in the paper. It is suggested to include it in the paper.

Reply: Thank you for the comment. The convergence analysis section has been added. Please see Section 5.

5. Authors solved very simple numerical examples (special case of equation (3)) using the proposed method to show the applicability of present method. It is suggested to solve more challenging diffusion problems in order to demonstrate the wider applicability of the method. **Reply:** Thank you for the comment. The numerical solution of a more challenging diffusion problem will be investigated further and will be published nearly in the future.

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Reply: Thank you for the comment. The above questions have been answered accordingly. Please see Page 2, Line 83-95.



10 Agustus 2021 09.21

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Dear Dr. Sunarto,

Congratulations on the acceptance of your manuscript, and thank you for your interest in submitting your work to Fractal Fract:

Manuscript ID: fractalfract-1290842 Type of manuscript: Article Title: Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation Authors: Andang Sunarto *, Praveen Agarwal, Jumat Sulaiman, Jackel Vui Lung Chew *, Shaher Momani Received: 23 June 2021 E-mails: andang99@gmail.com, goyal.praveen2011@gmail.com, jumat@ums.edu.my, jackelchew93@ums.edu.my, S.Momani@ju.edu.jo

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Kind regards, Carlo Cattani Editor-in-Chief

Andang Sunarto <andang99@gmail.com> Kepada: Fractal Fract Editorial Office <fractalfract@mdpi.com>

Greeting Thank you very much for your information 10 Agustus 2021 09.28

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